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HW Three, MTH 418, Spring 2016

Ayman Badawi

QUESTION 1. (i) Let H be a circuit graph that is not a cycle. First show that H has at least 5 vertices. Then show that H must have a cycle (i.e., show that girth of H not = infinity)

(ii) Construct a graph H that is a circuit but not a cycle with exactly 7 edges. Find the girth of H.

(iii) Let H be a connected graph with diameter 3. Prove (in at most three lines) that $\overline{G}$ is connected.

(iv) Let H be a graph of order m, $d_1 = |E(H)|$, and $d_2 = |E(\overline{H})|$. Prove that $d_1 + d_2 = \frac{m^2-m}{2}$.

(v) Find the adjacency matrix of C_4, say A. Use a calculator and find the eigenvalues of A, say a_1, a_2, a_3, a_4 (there must be 4 eigenvalues but not necessarily distinct). Find $d = a_1^2 + ... + a_4^2$. What is the relation between d and the size of C_4. In fact, your conclusion is true if we let A be an adjacency matrix of a graph H (nothing special about C_4).

(vi) Let H be a graph with vertex-set $= \{v_1, ..., v_\}$ and $D = K_3$ with vertex-set $= \{w_1, w_2, w_3\}$, Let $F = H \times D$ (Graph Product). Hence $(v1, w_1), (v_2, w_3) \in V(F)$. Assume $v_1 - v_3 - v_5 - v_2$ is the shortest path (walk) in H from v_1 to v_2. Find the distance between $(v1, w_1)$ and (v_2, w_3). Construct a shortest path from $(v1, w_1)$ to (v_2, w_3).

(vii) Let $H = K_{3,3}$. Construct two graphs F, D such that F, D, H are non-isomorphic graphs but H, F, and D have the same associated non-increasing sequence on the degrees of the vertices.

(viii) Convince me that it is impossible to construct a graph of order 7 such that each vertex is of degree 5. Convince me it is possible to construct a graph of order 8 such that each vertex is of degree 6. If possible, Construct a connected graph of order 8 such that each vertex is of degree 5.

(ix) Give me an example of two graphs, each is of order 6, both have the same associated non-increasing sequence on the degrees of the vertices, but one of them is disconnected while the other is connected.

Due date: Sunday at noon March 20 ,2016 Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com